

# Dirac Structure of the Nucleus-Nucleus Potential in Heavy Ion Collisions

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## Abstract

We investigate nuclear matter properties in the relativistic Brueckner approach. The in-medium on-shell T-matrix is represented covariantly by five Lorentz invariant amplitudes from which we deduce directly the nucleon self-energy. To enforce correct Hartree-Fock results we develop a subtraction scheme which treats the bare nucleon-nucleon potential exactly in accordance to the different types of meson exchanges. For the higher order correlations we employ two different covariant representations in order to study the uncertainty inherent in the approach. The nuclear matter bulk properties are only slightly sensitive on the explicit representation used. However, we obtain new Coester lines for the various Bonn potentials which are shifted towards the empirical region of saturation.

## 1 Introduction

The investigation of nuclear matter properties within the relativistic Dirac-Brueckner-Hartree-Fock (DBHF) approach [1, 2, 3, 4, 5, 6] remains a fundamental topic in theoretical nuclear structure studies. Compared to non-relativistic approaches the relativistic DBHF treatment turned out to be a major step forward in the explanation of the saturation mechanism of nuclear matter. The saturation points obtained for non-relativistic calculations, throughout all possible choices of different nucleon-nucleon interactions, are located on the so called 'Coester line' [7] which does not meet the empirical saturation region. Using modern nucleon-nucleon interactions of the one-boson exchange type [8] the relativistic calculations also reveal such Coester lines which are, however, significantly shifted towards the empirical region [3].

On the other hand, many details of the relativistic theory are still not fully resolved. In particular, the precise form of the nucleon self-energy, i.e. the magnitude and the momentum dependence of the scalar and vector self-energy components are a question of current debate [3, 4, 5]. Since the self-energy describes the dressing of the particles inside the medium and thus determines the relativistic mean

field this fact states a severe problem. Different techniques to handle the DBHF problem can lead to significantly different results [3, 4, 5, 6]. In a recent work [5] we found that the momentum dependence of the nucleon self-energy is dominated by the one-pion exchange contribution which accounts for the nuclear tensor force.

Unfortunately, the treatment of the  $\pi NN$  vertex and the corresponding self-energy contributions is closely connected to a severe ambiguity in the T-matrix representation [5]. The DBHF approach starts from a realistic nucleon-nucleon potential of the one-boson exchange type, i.e. the Bonn potentials [9]. As for the free two-body scattering problem, anti-particle states are neglected, and thus one works exclusively with positive energy states. Hence, a direct determination of the nucleon self-energy operator is not possible since not all matrix elements of this operator are known. Horowitz and Serot have therefore developed a projection technique to determine the scalar and vector self-energy components from the in-medium T-matrix [1]. In this approach the T-matrix is represented covariantly by Dirac operators and Lorentz invariant amplitudes where the latter are determined from the positive-energy on-shell T-matrix elements.

The whole problem arises from the fact mentioned above, namely that one does not include negative energy states and therefore neglects the excitation of anti-nucleons. The inclusion of negative energy excitations with 4 states for each spinor yields  $4^4 = 256$  types of two-body matrix elements concerning their spinor structure. Symmetry arguments reduce this to 44 for on-shell particles. [10]. If one takes now only positive energy solutions into account this reduces to  $2^4 = 16$  two-body matrix elements. Considering in addition only on-shell matrix elements the number of independent matrix elements can be further reduced by symmetry arguments down to 5. Thus, all on-shell two-body matrix elements can be expanded into five Lorentz invariants. But these five invariants are not unique since the Dirac matrices involve always also negative energy states and thus a decomposition of the one-body nucleon-nucleon potential into a Lorentz scalar and a Lorentz vector contribution depends on the choice of these five Lorentz invariants mentioned above. The best choice would be to separate completely the negative energy Dirac states. But since this is not possible, there is not a unique but only an 'optimal choice'. The topic of this paper is the form of this 'optimal choice' of the five invariants.

The paper is organized as follows: In section 2 we briefly review the Dirac-Brueckner Hartree-Fock approach. In section 3 we introduce the projection technique and discuss the different covariant representations used for the on-shell T-matrix. Nuclear matter bulk properties are then discussed in section 4. At the end we summarize and conclude our work.

## 2 The relativistic Brueckner approach

## 2.1 The coupled set of equations

In the relativistic Brueckner approach the nucleon inside the nuclear medium is viewed as a dressed particle in consequence of its two-body interaction with the surrounding nucleons. The in-medium interaction of the nucleons is treated in the ladder approximation of the relativistic Bethe-Salpeter equation

$$T = V + i \int V Q G G T \quad , \quad (1)$$

where  $T$  denotes the T-matrix.  $V$  is the bare nucleon-nucleon interaction. The intermediate off-shell nucleons in the scattering equation are described by a two-particle propagator  $iGG$ . We replace this propagator by the Thompson propagator [11]. The Pauli operator  $Q$  in the Thompson equation accounts for the influence of the medium by the Pauli-principle and projects the intermediate scattering states out of the Fermi sea.

The Green's function  $G$  fulfills the Dyson equation

$$G = G_0 + G_0 \Sigma G \quad . \quad (2)$$

$G_0$  denotes the free nucleon propagator while the influence of the surrounding nucleons is expressed by the nucleon self-energy  $\Sigma$ . In Brueckner theory this self-energy is determined by summing up the interaction with all the nucleons inside the Fermi sea

$$\Sigma = -i \int_F (Tr[GT] - GT) \quad . \quad (3)$$

The Dirac structure of the self-energy in isospin saturated nuclear matter follows from translational and rotational invariance, parity conservation and time reversal invariance [12]. In the nuclear matter rest frame the self-energy has the simple form

$$\Sigma(k, k_F) = \Sigma_s(k, k_F) - \gamma_0 \Sigma_o(k, k_F) + \boldsymbol{\gamma} \cdot \mathbf{k} \Sigma_v(k, k_F) \quad , \quad (4)$$

with  $k_\mu$  being the nucleon four-momentum. By taking the traces in Dirac space as [1, 4]

$$\Sigma_s = \frac{1}{4} tr [\Sigma] \quad , \quad \Sigma_o = \frac{-1}{4} tr [\gamma_0 \Sigma] \quad , \quad \Sigma_v = \frac{-1}{4|\mathbf{k}|^2} tr [\boldsymbol{\gamma} \cdot \mathbf{k} \Sigma] \quad (5)$$

one can calculate the different Lorentz components of the self-energy.

## 2.2 The in-medium T-matrix

We apply the relativistic Thompson equation [11] to solve the scattering problem of two nucleons in the nuclear medium. In the two-particle center of mass (c.m.) frame - the natural frame for studying the two-particle scattering process - this Thompson equation can be written as [2, 4]

$$\begin{aligned} T(\mathbf{p}, \mathbf{q}, x)|_{c.m.} &= V(\mathbf{p}, \mathbf{q}) \\ &+ \int \frac{d^3 \mathbf{k}}{(2\pi)^3} V(\mathbf{p}, \mathbf{k}) \frac{\tilde{m}_F^{*2}}{\tilde{E}^{*2}(\mathbf{k})} \frac{Q(\mathbf{k}, x)}{2\tilde{E}^*(\mathbf{q}) - 2\tilde{E}^*(\mathbf{k}) + i\epsilon} T(\mathbf{k}, \mathbf{q}, x) \quad , \end{aligned} \quad (6)$$

where  $\mathbf{q} = (\mathbf{q}_1 - \mathbf{q}_2)/2$  is the relative three-momentum of the initial state while  $\mathbf{k}$  and  $\mathbf{p}$  are the relative three-momenta of the intermediate and final states, respectively. The total four-momentum of the two-nucleon system is  $\tilde{P}^* = \tilde{q}_1^* + \tilde{q}_2^*$ .  $\sqrt{\tilde{s}^*} = 2\tilde{E}^*(\mathbf{q}) = 2\sqrt{\mathbf{q}^2 + \tilde{m}_F^{*2}}$  is the starting energy in (6). If  $\mathbf{q}_1$  and  $\mathbf{q}_2$  are nuclear matter rest frame momenta of the nucleons in the initial state, the boost-velocity  $\mathbf{u}$  into the c.m. frame is given by

$$\mathbf{u} = \mathbf{P}/\sqrt{\tilde{s}^* + \mathbf{P}^2} \quad , \quad (7)$$

with the total three-momentum and the invariant mass  $\mathbf{P} = \mathbf{q}_1 + \mathbf{q}_2$  and  $\tilde{s}^* = (\tilde{E}^*(\mathbf{q}_1) + \tilde{E}^*(\mathbf{q}_2))^2 - \mathbf{P}^2$ , respectively. In Eq. (7)  $x$  denotes the set of additional parameters  $x = \{k_F, \tilde{m}_F^*, |\mathbf{u}|\}$  on which the T-matrix depends.

Applying standard techniques as explained in detail by Erkelenz [8] we solve the Thompson equation in the c.m. frame and calculate the plane-wave helicity matrix elements of the T-matrix. The subspace of negative energy states is omitted in the current Brueckner approach. In this way we avoid the delicate problem of infinities in the theory which would generally appear if we would include contributions from negative energy nucleons in the Dirac sea [1, 6].

### 3 Covariant representations and the nucleon self-energy

#### 3.1 Pseudo-scalar representation

To use the trace formulas, Eqs. (5), one has to represent the T-matrix covariantly. A set of five linearly independent covariants is sufficient for such a T-matrix representation because on-shell only five helicity matrix elements appear as solution of the Thompson equation. A linearly independent although not unique set of five covariants is given by the Fermi covariants

$$S = 1 \otimes 1, V = \gamma^\mu \otimes \gamma_\mu, T = \sigma^{\mu\nu} \otimes \sigma_{\mu\nu}, A = \gamma_5 \gamma^\mu \otimes \gamma_5 \gamma_\mu, P = \gamma_5 \otimes \gamma_5. \quad (8)$$

Using this special set - the so called 'pseudo-scalar choice' - the on-shell T-matrix for definite isospin I can be represented covariantly as [1]

$$\begin{aligned} T^I(|\mathbf{p}|, \theta, x) &= F_S^I(|\mathbf{p}|, \theta, x)S + F_V^I(|\mathbf{p}|, \theta, x)V + F_T^I(|\mathbf{p}|, \theta, x)T \\ &\quad + F_A^I(|\mathbf{p}|, \theta, x)A + F_P^I(|\mathbf{p}|, \theta, x)P \quad . \end{aligned} \quad (9)$$

Here  $\mathbf{p}$  and  $\theta$  denote the relative three-momentum and the scattering angle between the scattered nucleons in the c.m. frame, respectively. Applying the covariant representation (9) for the on-shell T-matrix the nucleon self-energy in isospin saturated nuclear matter is evaluated to be [4]

$$\Sigma_{\alpha\beta}(k, k_F) = \int \frac{d^3\mathbf{q}}{(2\pi)^3} \frac{\theta(k_F - |\mathbf{q}|)}{\tilde{E}^*(\mathbf{q})} \left[ \tilde{m}_F^* 1_{\alpha\beta} F_S + \tilde{q}_{\alpha\beta}^* F_V \right] \quad , \quad (10)$$

where the isospin averaged amplitudes are defined as

$$F_i(|\mathbf{p}|, 0, x) := \frac{1}{2} [F_i^{I=0}(|\mathbf{p}|, 0, x) + 3F_i^{I=1}(|\mathbf{p}|, 0, x)] \quad . \quad (11)$$

In Fig. 1 we show the result of a self-consistent DBHF calculation for the nucleon self-energy in nuclear matter applying as representation for the on-shell T-matrix the *ps* representation (9). As bare interaction we have used the Bonn A potential [9] and, for comparison, the  $\sigma$ - $\omega$  model potential which was originally used by Horowitz and Serot [1]. The Fermi momentum is  $k_F = 1.34 fm^{-1}$ . As

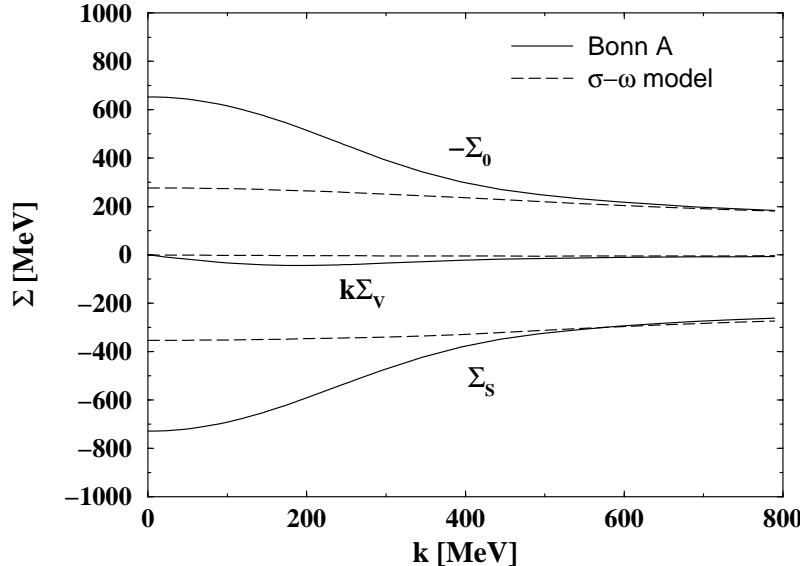


Figure 1: Momentum dependence of the DBHF nucleon self-energy components in nuclear matter at  $k_F = 1.34 fm^{-1}$  using as bare nucleon-nucleon potential Bonn A (solid) and the  $\sigma$ - $\omega$  model potential (dashed). For the T-matrix the *ps* representation (9) is applied.

already discussed in Ref. [4], we see a pronounced momentum dependence of the nucleon self-energy components with the full Bonn A while in the case of the  $\sigma$ - $\omega$  model potential the dependence on the momentum is rather weak. A strong momentum dependence leads to unphysical results deep inside the Fermi sea since the effective mass drops to values which are close to zero. Therefore in Ref. [5] the strong momentum dependence of the self-energy was studied in more detail and found to originate mainly from the one-pion exchange contribution to the self-energy.

### 3.2 Complete pseudo-vector representation

To suppress the undesirable pseudo-scalar contribution of the pion to the nucleon self-energy we have to use the 'complete' *pv* representation of the T-matrix [13]

$$\begin{aligned} T^I(|\mathbf{p}|, \theta, x) &= g_S^I(|\mathbf{p}|, \theta, x)S - g_{\bar{S}}^I(|\mathbf{p}|, \theta, x)\bar{S} + g_A^I(|\mathbf{p}|, \theta, x)(A - \bar{A}) \\ &\quad + g_{PV}^I(|\mathbf{p}|, \theta, x)PV - g_{\bar{PV}}^I(|\mathbf{p}|, \theta, x)\bar{PV} \quad . \end{aligned} \quad (12)$$

The amplitudes  $g^I(\theta)$  are explicitly given in [14]. In Fig. 2 we present the full self-consistent DBHF calculation with the 'complete' *pv* representation of the T-matrix [5]. The DBHF nucleon self-energy

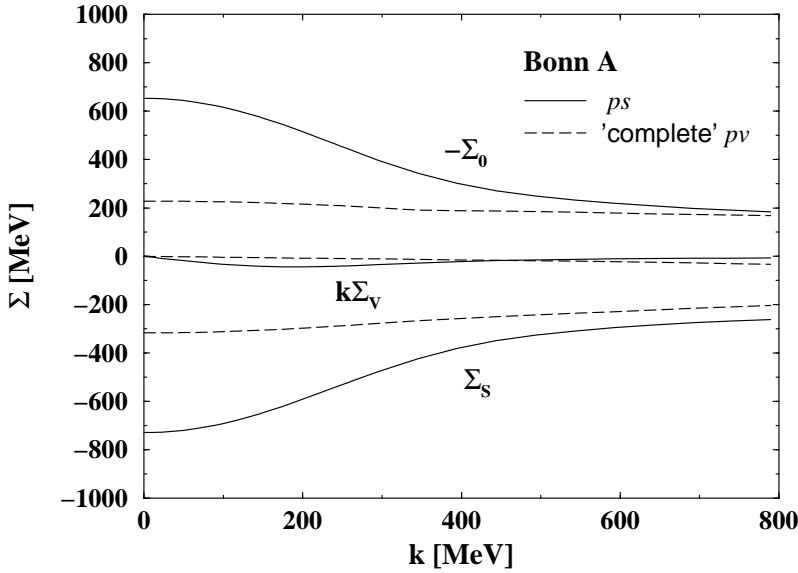


Figure 2: Momentum dependence of the DBHF nucleon self-energy components in nuclear matter at  $k_F = 1.34 \text{ fm}^{-1}$  using Bonn A as bare nucleon-nucleon interaction. For the T-matrix the *ps* representation (Eq. (9), solid) and the 'complete' *pv* representation (Eq. (12), dashed) are applied.

components are indeed weakly momentum dependent. The single-pion exchange contribution to the interaction, which was previously dominating at low nucleon momenta, is now strongly suppressed. Consequently the result within the 'complete' *pv* representation using the Bonn A potential resembles the result within the  $\sigma$ - $\omega$  model potential, see Fig. 1 where the *ps* representation was used. To suppress the pion contribution to the in-medium T-matrix a correct pseudo-vector like covariant representation is essential for the calculation of the nucleon self-energy in nuclear matter.

### 3.3 Covariant representations of the subtracted T-matrix

The 'complete' *pv* representation successfully reproduces the HF nucleon self-energy in the case of the pion exchange. However, as already pointed out in [5], the 'complete' *pv* representation fails to reproduce the HF nucleon self-energy if other meson exchange potentials are applied as bare interaction. Hence, it should be reasonable to treat the bare interaction and the higher order ladder graphs of the meson exchange potential separately. Since the single-meson exchange potential is actually known analytically we can represent it covariantly by a mixed representation of the form

$$V = V_{\pi,\eta}^{PV} + V_{\sigma,\omega,\rho,\delta}^P . \quad (13)$$

Here the  $\pi$ - and  $\eta$ -meson contributions are treated as pseudo-vector while for the  $(\sigma, \omega, \rho, \delta)$ -meson contributions of the Bonn potential the *ps* representation is applied. The higher order ladder diagrams of the T-matrix

$$T_{Sub} = T - V = i \int V Q G G T = \sum_{n=1}^{\infty} \int V (i Q G G V)^n , \quad (14)$$

in the following called the subtracted T-matrix, can not be represented correctly in a mixed form since we can not disentangle the different meson contributions to this part of the full in-medium interaction. The representation of the subtracted T-matrix remains therefore ambiguous. However, if the pion exchange dominantly contributes to the Hartree-Fock level a *ps* representation of the subtracted T-matrix should be more appropriate because then the higher order contributions of other meson exchange potentials are not treated incorrectly as pseudo-vector. Thus the most favorable representation of the T-matrix is given by the *ps* representation

$$T^P = T_{Sub}^P + V_{\pi,\eta}^{PV} + V_{\sigma,\omega,\rho,\delta}^P \quad . \quad (15)$$

Here the *ps* representation for  $T_{Sub}^P$  is determined via the matrix elements

$$\langle \mathbf{p} \lambda'_1 \lambda'_2 | T_{Sub}^I(x) | \mathbf{q} \lambda_1 \lambda_2 \rangle := \langle \mathbf{p} \lambda'_1 \lambda'_2 | T^I(x) - V^I(x) | \mathbf{q} \lambda_1 \lambda_2 \rangle \quad , \quad (16)$$

with subsequently applying the projection scheme as in Eq. (??). An alternative representation of the T-matrix is given by the *pv* representation

$$T^{PV} = T_{Sub}^{PV} + V_{\pi,\eta}^{PV} + V_{\sigma,\omega,\rho,\delta}^P \quad , \quad (17)$$

where the subtracted T-matrix is represented by the 'complete' *pv* representation (12). This representation is similar to the 'complete' *pv* representation of the full T-matrix, however, with the advantage that now the pseudo-scalar contributions in the bare nucleon-nucleon interaction, e.g. the single-omega exchange potential, are represented correctly. In the next section we will use both representations, (15) and (17), to study the properties of nuclear matter in the DBHF approach. In this way we can determine the influence of the higher order ladder graphs to the in-medium interaction in a more quantitative way. Furthermore, these two representations set the range of the remaining ambiguity concerning the representation of the T-matrix, i.e. after separating the leading order contributions.

## 4 The equation-of-state of nuclear matter

In the relativistic Brueckner theory the energy per particle is defined as the kinetic plus half the potential energy

$$E/A = \frac{1}{\rho} \sum_{\mathbf{k},\lambda} \langle \bar{u}_\lambda(\mathbf{k}) | \gamma \cdot \mathbf{k} + M + \frac{1}{2} \Sigma(k) | u_\lambda(\mathbf{k}) \rangle \frac{\tilde{m}^*(k)}{\tilde{E}^*(k)} - M \quad . \quad (18)$$

In Fig. 3 we show the binding energy per particle  $E/A$  as a function of the density, calculated with Bonn A, B and C. For the T-matrix the subtraction scheme with the *ps* representation (15) is applied. With Bonn A one can reproduce the empirical saturation point of nuclear matter, shown as shaded region in the figure. The other Bonn potentials give less binding energy although the saturation density

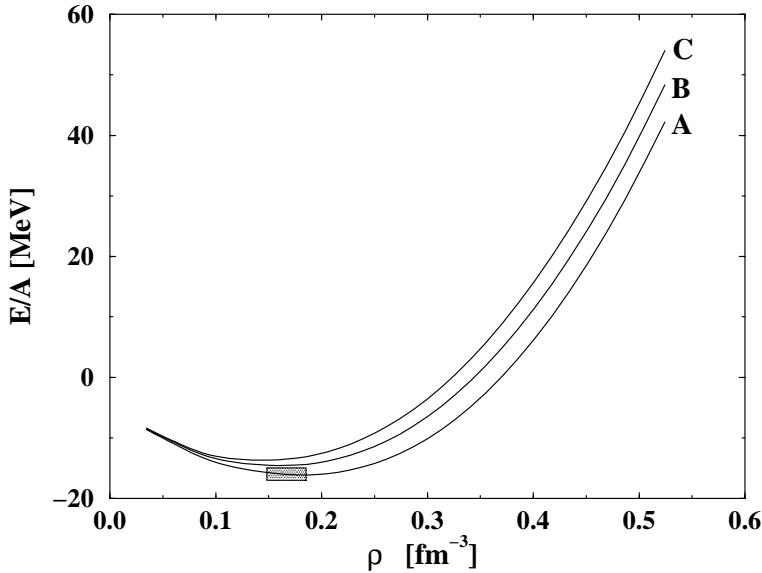


Figure 3: Binding energy per particle as a function of nuclear matter density. As bare nucleon-nucleon interaction the potentials Bonn A, B and C are used. For the T-matrix the subtraction scheme with the  $ps$  representation (15) is applied. The shaded box denotes the empirical saturation region of nuclear matter.

is always close to the empirically known value. The result for the binding energy per particle using the two representations (15) and (17) for the T-matrix are very similar. At saturation density the binding energy is only 0.5 MeV smaller using the pseudo-vector representation of the subtracted T-matrix. Thus, the energy per particle is not very sensitive on the explicit representation of the subtracted T-matrix.

The present results are summarized in Fig. 4 where the corresponding saturation points for the three different versions of the Bonn potential are shown. We compare the results with the two representation of the subtracted T-matrix with the results of the calculation of Brockmann and Machleidt (BM), Ref. [3]. With the improved representation schemes (15) and (17) for the T-matrix one obtains new 'Coester-lines' which are left of the original one, i.e. shifted towards the empirical region. The refined treatment of the T-matrix representation leads to an enhancement of the binding energy connected with a reduced saturation density. As in the previous calculations, Bonn A is still the only one which meets the empirical region. However, due to an increased binding energy Bonn B is now much closer to empirical region. This observation is consistent with the present treatment. The different types of Bonn potentials essentially vary in the strength of the nuclear tensor force determined by the  $\pi NN$  form factor. Bonn A which has the smallest tensor force yields the smallest D-state probability of the deuteron and only a pure description of the  $^3D_1$  phase shift [3, 9]. Thus it appears that a refined treatment of the pion exchange leads to improved nuclear matter results for the more realistic Bonn B potential. Bonn C, however, is still far off the empirical region.

Furthermore, it can be seen from Fig. (4) that the final nuclear matter bulk properties depend

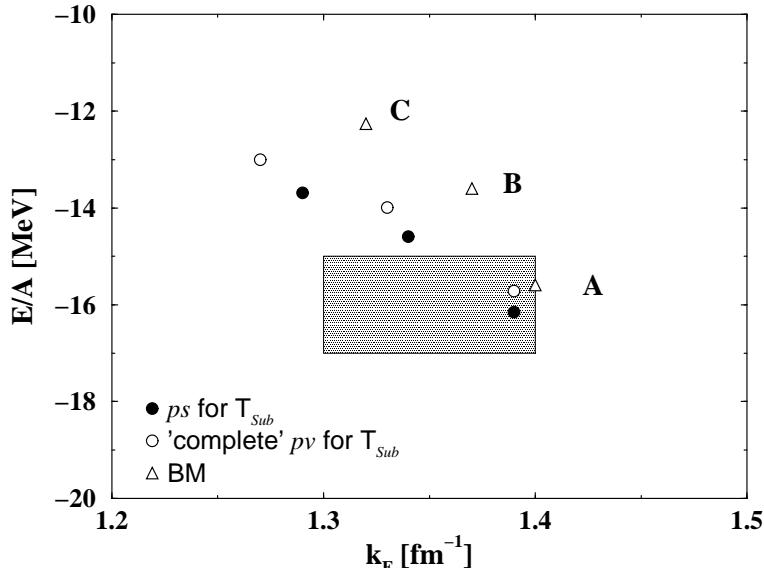


Figure 4: Saturation points of nuclear matter. As bare nucleon-nucleon interaction the Bonn potentials A,B and C are used. For the T-matrix the subtraction scheme with the  $ps$  representation (Eq. (15), filled circles) and the  $pv$  representation (Eq. (17), open circles) are applied. As open triangles the results of the calculation of Brockmann and Machleidt (BM), Ref. [3], are shown.

only moderate on the representation of the subtracted T-matrix. In Ref. [5] we tried already to determine the range of inherent uncertainty in the relativistic Brueckner approach which is due to the ambiguities concerning the representation of the T-matrix discussed in Section 3. By the separate treatment of the Born contribution to the T-matrix we end up now with a much narrower uncertainty band which is given by the  $ps$  or complete  $pv$  representation of the ladder kernel, i.e. the subtracted T-matrix. Over the different types of Bonn interactions the two methods lead to a variation of 0.5 MeV in the binding energy,  $0.1\text{--}0.2\text{ fm}^{-1}$  in the Fermi momentum, and to about 30 MeV concerning the value of the effective mass at saturation density. The values for incompressibility are also close in the two approaches, i.e. they differ by less than 10 MeV. Within the  $ps$  representation of the subtracted T-matrix, Bonn B and C now yield very small kompression moduli around  $K = 150\text{MeV}$  and  $K = 115\text{MeV}$ , respectively. For Bonn A a value of  $K = 230\text{MeV}$  is obtained. This value agrees with the empirical value of the kompression modulus of  $K = 210 \pm 30\text{MeV}$ . Here Brockmann and Machleidt found much larger values for all three Bonn potentials.

## 5 Summary

We have investigated the nuclear matter properties in the relativistic Brueckner approach. The required representation of the T-matrix by Lorentz invariant amplitudes suffers thereby from on-shell ambiguities concerning the pseudo-scalar or pseudo-vector nature of the interaction. We minimized this ambiguity by separating the leading order, i.e. the single-meson exchange, from the full T-matrix.

The remaining higher order correlations, i.e. the ladder kernel, are then represented either completely as pseudo-scalar or as pseudo-vector.

As a major result of our investigation we obtain new 'Coester lines' for the various Bonn potentials. Compared to previous treatments these are shifted towards the empirically known saturation point. Bonn A is still the only potential which really meets the empirical region of saturation, but, with improved saturation properties compared to previous treatments. The refined treatment of the pion exchange leads on the other hand also to improved results for the – from the view of the phase shift analysis – more realistic Bonn B potential. Furthermore, we found that the equation-of-state is strongly softened compared to previous calculations. Actually with Bonn A we obtain a compression modulus of  $K \sim 230\text{MeV}$  which is in good agreement with the empirical value.

To summarize our results, we obtained new results for the nuclear matter properties within the projection technique employing a new method for the T-matrix representation. The final results are at lower densities almost insensitive on the explicit choice made for the representation. However, at higher densities, certain differences occur when using different representation schemes. We want to stress that the ambiguity in the projection technique is still not fully resolved yet. We plan to look on off-shell T-matrix elements in the future since off-shell matrix elements of the pseudo-scalar and pseudo-vector covariants differ significantly. We hope that this might bring more insight on what is the correct on-shell representation of the T-matrix.

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